

Study on the $\Upsilon(1S) \rightarrow B_c M$ weak decays

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Abstract

Motivated by the prospects of the potential $\Upsilon(1S)$ particle at high-luminosity heavy-flavor experiments, we studied the $\Upsilon(1S) \rightarrow B_c M$ weak decays, where $M = \pi, \rho, K^{(*)}$. The nonfactorizable contributions to hadronic matrix elements are taken into consideration with the QCDF approach. It is found that the CKM-favored $\Upsilon(1S) \rightarrow B_c \rho$ decay has branching ratio of $\mathcal{O}(10^{-10})$, which might be measured promisingly by the future experiments.

PACS numbers: 13.25.Gv 12.39.St 14.40.Pq

Keywords: Υ meson; weak decay; QCD factorization

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I. INTRODUCTION

The first evidence for upsilons, the bound states of $b\bar{b}$, was observed in collisions of protons with a stationary nuclear target at Fermilab in 1977 [1, 2]. From that moment on, bottomonia have been a subject of intensive experimental and theoretical research. Some of the salient features of upsilons are as follows [3]: (1) In the upsilons rest frame, the relative motion of the b quark is sufficiently slow. Nonrelativistic Schrödinger equation can be used to describe the spectrum of bottomonia states and thus one can learn about the interquark binding forces. (2) The $\Upsilon(nS)$ particles, with the radial quantum number $n = 1, 2$ and 3 , decay primarily via the annihilation of the $b\bar{b}$ quark pairs into three gluons. Thus the properties of the invisible gluons and of the gluon-quark coupling can be gleaned through the study of the upsilons decay. (3) Compared with the u, d, s light quarks, the relatively large mass of the heavy b quark implies a nonnegligible coupling to the Higgs bosons, making upsilons to be one of the best hunting grounds for light Higgs particles. By now, our knowledge of the properties of bottomonia comes primarily from e^+e^- annihilation.

The $\Upsilon(1S)$ particle is the ground state of the vector bottomonia with quantum number of $I^G J^{PC} = 0^- 1^{--}$ [4]. The mass of the $\Upsilon(1S)$ particle, $m_{\Upsilon(1S)} = 9460.30 \pm 0.26$ [4], is about three times heavier than the mass of the J/ψ particle (the ground state of charmonia with the same quantum number of $I^G J^{PC}$). On one hand, compared with the J/ψ decay, much richer decay channels could be accessed by the $\Upsilon(1S)$ particle. On the other hand, the coupling constant α_s for the $\Upsilon(1S)$ decay is smaller than that for the J/ψ decay due to the QCD nature of asymptotic freedom, which results in hadronic partial width $\Gamma(\Upsilon(1S) \rightarrow ggg) < \Gamma(J/\psi \rightarrow ggg)$, although the possible phase space in the $\Upsilon(1S)$ decay is larger than that in the J/ψ decay. In addition, the squared value of the b quark charge, $Q_b^2 = 1/9$, is less than that of the c quark charge, $Q_c^2 = 4/9$, which results in electromagnetic partial width $\Gamma(\Upsilon(1S) \rightarrow \gamma^*) < \Gamma(J/\psi \rightarrow \gamma^*)$. So the full decay width of the $\Upsilon(1S)$ particle, $\Gamma_{\Upsilon(1S)} = 54.02 \pm 1.25$ keV, is less than that of the J/ψ particle, $\Gamma_{J/\psi} = 92.9 \pm 2.8$ keV [4]. Furthermore, one of the outstanding properties of all upsilons below $B\bar{B}$ threshold is their narrow decay width of tens of keV [4].

The $\Upsilon(1S)$ and J/ψ particles share the similar decay mechanism. As is the case for the J/ψ particle, strong decays of the $\Upsilon(1S)$ particle are suppressed by the phenomenological OZI (Okubo-Zweig-Iizuka) rules [5–7], so electromagnetic interactions and radiative transi-

tions become competitive. It is expected that, at the lowest order approximation, the decay modes of the $\Upsilon(1S)$ particle could be subdivided into four types: (1) The lion's share of the decay width is the hadronic decay via the annihilation of the $b\bar{b}$ quark pairs into three gluons, i.e., some $(81.7\pm 0.7)\%$ via $\Upsilon(1S) \rightarrow ggg$ [4]. (2) The partial width of the electromagnetic decay via the annihilation of the $b\bar{b}$ quark pairs into a virtual photon could be written approximately as $(3+R)\Gamma_{\ell\ell}$, where the value of R is the ratio of inclusive production of hadrons to the $\mu^+\mu^-$ pair production rate at the energy scale of $m_{\Upsilon(1S)}$, and $\Gamma_{\ell\ell}$ is the partial width of decay into dileptons. (3) Branching ratio of the radiative decay is about $Br(\Upsilon(1S)\rightarrow\gamma gg) \simeq (2.2\pm 0.6)\%$ [4]. The up-to-date research for light Higgs bosons in the $\Upsilon(1S)$ radiative decay has been performed by CLEO [8], Belle[9] and BaBar [10] Collaborations. (4) The magnetic dipole transition decay, $\Upsilon(1S) \rightarrow \gamma\eta_b(1S)$, is very challenging to experimental physicist due to the very soft photon and pollution from other processes, such as $\Upsilon(1S) \rightarrow \pi^0 X \rightarrow \gamma\gamma X$ [11]. The experimental signal for $\Upsilon(1S) \rightarrow \gamma\eta_b(1S)$ has not been discovered until now. Besides, the $\Upsilon(1S)$ particle could also decay via the weak interactions, although the branching ratio for a single b or \bar{b} quark decay is tiny, about $2/\tau_B\Gamma_{\Upsilon(1S)} \sim 10^{-8}$ [4]. In this paper, we will estimate the branching ratios for the flavor-changing nonleptonic $\Upsilon(1S) \rightarrow B_c M$ weak decays with the QCD factorization (QCDF) approach [12, 13], where $M = \pi, \rho, K$ and K^* . The motivation is listed as follows.

TABLE I: Summary of data samples (in the unit of 10^6) of the $\Upsilon(nS)$ particles below $B\bar{B}$ threshold collected by Belle, BaBar and CLEO Collaborations. The data in the 5th column correspond to total number of the $\Upsilon(1S)$ particle, including events from hadronic dipion transitions between upsilons $\Upsilon(2S, 3S) \rightarrow \pi\pi\Upsilon(1S)$, where branching ratios for the dipion decays $\Upsilon(2S) \rightarrow \pi^+\pi^-\Upsilon(1S)$, $\pi^0\pi^0\Upsilon(1S)$ and $\Upsilon(3S) \rightarrow \pi^+\pi^-\Upsilon(1S)$, $\pi^0\pi^0\Upsilon(1S)$ are $(17.85\pm 0.26)\%$, $(8.6\pm 0.4)\%$, $(4.37\pm 0.08)\%$ and $(2.20\pm 0.13)\%$, respectively [4].

| | $\Upsilon(1S)$ | $\Upsilon(2S)$ | $\Upsilon(3S)$ | total $\Upsilon(1S)$ |
|------------|----------------|----------------|----------------|----------------------|
| Belle [14] | 102 | 158 | 11 | 145 |
| BaBar [15] | | 121.8 | 98.6 | 39 |
| CLEO [16] | 22.78 | 9.45 | 8.89 | 26 |

From the experimental point of view, (1) there is plenty of upsilons at the high-luminosity dedicated bottomonia factories, for example, over 10^8 $\Upsilon(1S)$ at Belle (see Table I). Upsilons

are also observed by the on-duty ALICE [17], ATLAS [18], CMS [19], LHCb [20] experiments at LHC. It is hopefully expected that more upsilons could be accumulated with great precision at the running upgraded LHC and forthcoming SuperKEKB. The huge $\Upsilon(1S)$ data samples will provide good opportunities to search for the $\Upsilon(1S)$ weak decays which in some cases might be detectable. Theoretical studies on the $\Upsilon(1S)$ weak decays are just necessary to offer a ready reference. (2) For the two-body $\Upsilon(1S) \rightarrow B_c M$ decay, final states with opposite charges have definite energies and momenta in the center-of-mass frame of the $\Upsilon(1S)$ particle. Particularly, identification of a single charged B_c meson in the final state, which is free from inefficiently double tagging of the bottomed hadron pairs occurring above the $B\bar{B}$ threshold, would provide an unambiguous signature of the $\Upsilon(1S)$ weak decay. Of course, small branching ratios make the observation of the $\Upsilon(1S)$ weak decays extremely difficult, and evidences of an abnormally large production rate of single B_c mesons in the $\Upsilon(1S)$ decay might be a hint of new physics beyond the standard model.

From the theoretical point of view, the bottom-changing upilon weak decay could permit overconstraining parameters obtained from B meson decay, but few studies devoted to the nonleptonic upilon weak decay in the past. For the $\Upsilon(1S) \rightarrow B_c M$ decay, the amplitude is usually treated as the factorizable product of two independent factors: one describing the transition between heavy quarkonium $\Upsilon(1S)$ and B_c , and the other depicting the production of the M state from the vacuum. Previous works, such as Ref. [21] based on the spin symmetry and nonrecoil approximation, Ref. [22] based on the heavy quark effective theory, and Ref. [23] based on the Bauer-Stech-Wirbel (BSW) model [24], concentrated mainly upon the $\Upsilon(1S) \rightarrow B_c$ transition form factors which are related to the space integrals of the meson wave functions. As is well known, there exist hierarchical scales with nonrelativistic quantum chromodynamics (NRQCD) [25–27] which is an approach to deal with the heavy quarkonium, i.e. $M^2 \gg (Mv)^2 \gg (Mv^2)^2$, where M is the mass of heavy quark with typical velocities $v \sim \alpha_s \ll 1$. The physical contributions at scales of $Mv^2 \sim \Lambda_{\text{QCD}}$ are absorbed into wave functions of the $\Upsilon(1S)$ and B_c particles, thus the $\Upsilon(1S) \rightarrow B_c$ transition should be dominated by the nonperturbative dynamics. The nonfactorizable contributions above scales of Mv^2 has not been taken seriously in previous works. About 2000, M. Beneke *et al.* proposed the QCDF approach [12, 13], where nonfactorizable contributions could be estimated systematically with the perturbation theory based on collinear factorization approximation and power counteracting rules in the heavy quark limit [13], and the QCDF approach has been widely

applied to nonleptonic B meson decays. So it should be very interesting to study the $\Upsilon(1S) \rightarrow B_c M$ weak decays by considering nonfactorizable contributions with the attractive QCDF approach.

This paper is organized as follows. In section II, we will present the theoretical framework and the amplitudes for the nonleptonic two-body $\Upsilon(1S) \rightarrow B_c M$ weak decays with the QCDF approach. Section III is devoted to numerical results and discussion. The last section is our summary.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

The low energy effective Hamiltonian responsible for the $\Upsilon(1S) \rightarrow B_c M$ decays is [28]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V_{uq}^* \{C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu)\} + \text{h.c.}, \quad (1)$$

where the Fermi coupling constant $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [4]; Using the Wolfenstein parameterization, the Cabibbo-Kobayashi-Maskawa (CKM) factors can be expanded as a power series in the small parameter $\lambda = 0.22537(61)$ [4],

$$V_{cb} V_{ud}^* = A\lambda^2 - \frac{1}{2}A\lambda^4 - \frac{1}{8}A\lambda^6 + \mathcal{O}(\lambda^8), \quad (2)$$

$$V_{cb} V_{us}^* = A\lambda^3 + \mathcal{O}(\lambda^8). \quad (3)$$

The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above scales of μ . The Wilson coefficients C_i are calculable with the perturbation theory and have properly been evaluated to the next-to-leading order (NLO) with the renormalization group (RG) equation. The numerical values of the Wilson coefficients $C_{1,2}$ at scales of $\mu \sim \mathcal{O}(m_b)$ in naive dimensional regularization scheme are listed in Table II. The local tree four-quark operators are defined as follows.

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (4)$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (5)$$

where α and β are color indices and the sum over repeated indices is understood. To obtain the decay amplitudes, the remaining and the most intricate works are to calculate accurately hadronic matrix elements of local operators.

B. Hadronic matrix elements

Phenomenologically, the simplest treatment with hadronic matrix elements of four-quark operators is approximated by the product of two current matrix elements with color transparency ansatz [29] and naive factorization (NF) scheme [30, 31], and current matrix elements are further parameterized by decay constants and transition form factors. For example, previous studies on the $\Upsilon(1S) \rightarrow B_c M$ decay [22, 23] were based on NF approach.

As is well known, NF's defect is the disappearance of the renormalization scale dependence, the strong phases and the nonfactorizable contributions from hadronic matrix elements, resulting in nonphysical amplitudes and no CP violating asymmetries. To remedy NF's deficiencies, M. Beneke *et al.* proposed that hadronic matrix elements could be written as the convolution integrals of hard scattering kernels and light cone distribution amplitudes with the QCDF approach [12, 13].

For the $\Upsilon(1S) \rightarrow B_c M$ decay, the spectator quark is the heavy bottom (anti)quark. According to the QCDF's power counting rules [13], contributions from the spectator scattering are power suppressed. With the QCDF master formula, hadronic matrix elements could be written as :

$$\langle B_c M | Q_i | \Upsilon(1S) \rangle = \sum_i F_i^{\Upsilon \rightarrow B_c} \int dx H_i(x) \phi_M(x), \quad (6)$$

where transition form factor $F_i^{\Upsilon \rightarrow B_c}$ and light cone distribution amplitudes $\phi_M(x)$ of the emitted meson M are nonperturbative input parameters, hard scattering kernels $H_i(x)$ are computable order by order with the perturbation theory in principle.

The leading twist two-valence-particle distribution amplitudes of pseudoscalar and longitudinally polarized vector meson are defined in terms of Gegenbauer polynomials [32, 33]:

$$\phi_M(x) = 6 x \bar{x} \sum_{n=0}^{\infty} a_n^M C_n^{3/2}(x - \bar{x}), \quad (7)$$

where $\bar{x} = 1 - x$; a_n^M is the Gegenbauer moment and $a_0^M \equiv 1$.

After calculation, the decay amplitudes could be written as

$$\mathcal{A}(\Upsilon(1S) \rightarrow B_c M) = \langle B_c M | \mathcal{H}_{\text{eff}} | \Upsilon(1S) \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* a_1 \langle M | J^\mu | 0 \rangle \langle B_c | J_\mu | \Upsilon(1S) \rangle. \quad (8)$$

The coefficient a_1 in Eq.(8), including nonfactorizable contributions from QCD radiative vertex corrections, is written as [34]:

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N_c} C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} C_2^{\text{LO}} V. \quad (9)$$

For the transversely polarized vector meson, the factor V is zero beyond leading twist (twist-2) contributions. For the pseudoscalar and longitudinally polarized vector meson, with the modified minimal subtraction ($\overline{\text{MS}}$) scheme, the factor V is written as [34]:

$$V = 3 \log\left(\frac{m_b^2}{\mu^2}\right) + 3 \log\left(\frac{m_c^2}{\mu^2}\right) - 18 + \int_0^1 dx T(x) \phi_M(x), \quad (10)$$

where

$$\begin{aligned} T(x) = & \frac{c_a}{1-c_a} \log(c_a) - \frac{4c_b}{1-c_b} \log(c_b) \\ & + \frac{c_d}{1-c_d} \log(c_d) - \frac{4c_c}{1-c_c} \log(c_c) \\ & - r_c \left\{ \frac{c_a}{(1-c_a)^2} \log(c_a) + \frac{1}{1-c_a} \right\} \\ & - r_c^{-1} \left\{ \frac{c_d}{(1-c_d)^2} \log(c_d) + \frac{1}{1-c_d} \right\} \\ & + f(c_a) - f(c_b) - f(c_c) + f(c_d) \\ & + 2 \log(r_c^2) \{ \log(c_a) - \log(c_b) \}, \end{aligned} \quad (11)$$

$$f(c) = 2 \text{Li}_2\left(\frac{c-1}{c}\right) - \log^2(c) - \frac{2c}{1-c} \log(c), \quad (12)$$

and the relations

$$r_c = m_c/m_b, \quad (13)$$

$$c_a = x(1-r_c^2), \quad (14)$$

$$c_b = \bar{x}(1-r_c^2), \quad (15)$$

$$c_c = -c_a/r_c^2, \quad (16)$$

$$c_d = -c_b/r_c^2. \quad (17)$$

The numerical values of coefficient a_1 at scales of $\mu \sim \mathcal{O}(m_b)$ are listed in Table II.

C. Decay constants and form factors

The matrix elements of current operators are defined as follows:

$$\langle P(p) | A_\mu | 0 \rangle = -i f_P p_\mu, \quad (18)$$

$$\langle V(p, \epsilon) | V_\mu | 0 \rangle = f_V m_V \epsilon_{V,\mu}^*, \quad (19)$$

TABLE II: The numerical values of the Wilson coefficients $C_{1,2}$ and a_1 for the $\Upsilon(1S) \rightarrow B_c \pi$ decay at different scales, where $m_b = 4.78$ GeV [4].

| μ | NLO | | LO | | QCDF | |
|-----------|-------|--------|-------|--------|------------------|------------------|
| | C_1 | C_2 | C_1 | C_2 | $\text{Re}(a_1)$ | $\text{Im}(a_1)$ |
| $0.5 m_b$ | 1.128 | -0.269 | 1.168 | -0.337 | 1.076 | +0.027 |
| m_b | 1.076 | -0.173 | 1.110 | -0.235 | 1.054 | +0.015 |
| $1.5 m_b$ | 1.054 | -0.128 | 1.085 | -0.188 | 1.043 | +0.011 |
| $2.0 m_b$ | 1.041 | -0.100 | 1.070 | -0.159 | 1.036 | +0.008 |

where f_P and f_V are the decay constants of pseudoscalar and vector mesons, respectively; m_V and ϵ_V denote the mass and polarization of vector meson, respectively.

The transition form factors are defined as follows [22–24]:

$$\begin{aligned}
& \langle B_c(p_2) | V_\mu - A_\mu | \Upsilon(p_1, \epsilon) \rangle \\
&= -\epsilon_{\mu\nu\alpha\beta} \epsilon_\Upsilon^\nu q^\alpha (p_1 + p_2)^\beta \frac{V^{\Upsilon \rightarrow B_c}(q^2)}{m_\Upsilon + m_{B_c}} - i \frac{2 m_\Upsilon \epsilon_\Upsilon \cdot q}{q^2} q_\mu A_0^{\Upsilon \rightarrow B_c}(q^2) \\
&\quad - i \epsilon_{\Upsilon, \mu} (m_\Upsilon + m_{B_c}) A_1^{\Upsilon \rightarrow B_c}(q^2) - i \frac{\epsilon_\Upsilon \cdot q}{m_\Upsilon + m_{B_c}} (p_1 + p_2)_\mu A_2^{\Upsilon \rightarrow B_c}(q^2) \\
&\quad + i \frac{2 m_\Upsilon \epsilon_\Upsilon \cdot q}{q^2} q_\mu A_3^{\Upsilon \rightarrow B_c}(q^2),
\end{aligned} \tag{20}$$

where $q = p_1 - p_2$; and $A_0(0) = A_3(0)$ is required compulsorily to cancel singularities at the pole $q^2 = 0$. There is a relation among these form factors

$$2m_\Upsilon A_3(q^2) = (m_\Upsilon + m_{B_c}) A_1(q^2) + (m_\Upsilon - m_{B_c}) A_2(q^2). \tag{21}$$

It is clearly seen that there are only three independent form factors, $A_{0,1}(0)$ and $V(0)$, at the pole $q^2 = 0$ for the $\Upsilon(1S) \rightarrow B_c M$ decays. The form factors at the pole $q^2 = 0$ could be written as the overlap integrals of wave functions of mesons [24], i.e.,

$$A_0^{\Upsilon \rightarrow B_c}(0) = \int d\vec{k}_\perp \int_0^1 dx \left\{ \Phi_\Upsilon(k_\perp, x, 1, 0) \sigma_z \Phi_{B_c}(k_\perp, x, 0, 0) \right\}, \tag{22}$$

$$A_1^{\Upsilon \rightarrow B_c}(0) = \frac{m_b + m_c}{m_{\Upsilon(1S)} + m_{B_c}} I^{\Upsilon \rightarrow B_c} \tag{23}$$

$$V^{\Upsilon \rightarrow B_c}(0) = \frac{m_b - m_c}{m_{\Upsilon(1S)} - m_{B_c}} I^{\Upsilon \rightarrow B_c}, \tag{24}$$

$$I^{\Upsilon \rightarrow B_c} = \sqrt{2} \int d\vec{k}_\perp \int_0^1 \frac{dx}{x} \left\{ \Phi_\Upsilon(k_\perp, x, 1, -1) i\sigma_y \Phi_{B_c}(k_\perp, x, 0, 0) \right\}, \quad (25)$$

where $\sigma_{y,z}$ is a Pauli matrix acting on the spin indices of the decaying bottom quark; x and \vec{k}_\perp denote the fraction of the longitudinal momentum and the transverse momentum carried by the nonspectator quark, respectively.

Using the separation of momentum and spin variables, the wave functions of mesons can be written as

$$\Phi(\vec{k}_\perp, x, j, j_z) = \phi(\vec{k}_\perp, x) |s, s_z, s_1, s_2\rangle, \quad (26)$$

with the normalization condition,

$$\sum_{s_1, s_2} \int d\vec{k}_\perp \int_0^1 dx |\Phi(\vec{k}_\perp, x, j, j_z)|^2 = 1, \quad (27)$$

where $s_{1,2}$ denote the spin of valence quark in meson; $\vec{s} = \vec{s}_1 + \vec{s}_2$; $s = 1$ and 0 for the $\Upsilon(1S)$ and B_c particles, respectively.

For the ground states of heavy quarkonia $\Upsilon(1S)$ and B_c particles, we will take the solution of the Schödinger equation with nonrelativistic three-dimensional scalar harmonic oscillator potential,

$$\phi(\vec{k}) \sim \int d\vec{r} \phi(\vec{r}) e^{-i\vec{k}\cdot\vec{r}} \sim \int d\vec{r} e^{-\beta^2 r^2/2} e^{-i\vec{k}\cdot\vec{r}} \sim e^{-\vec{k}^2/2\beta^2}, \quad (28)$$

where the parameter β determines the average transverse quark momentum, i.e., $\langle \vec{k}_\perp^2 \rangle = \beta^2$. According to the power counting rules of NRQCD [25], the characteristic magnitude of the moment is order of Mv and $v \sim \alpha_s$. So we will take $\beta = M\alpha_s$ in our calculation. Employing the substitution ansatz [35],

$$\vec{k}^2 \rightarrow \frac{1}{4} \left\{ \frac{\vec{k}_\perp^2 + m_1^2}{x_1} + \frac{\vec{k}_\perp^2 + m_2^2}{x_2} + \frac{(m_1^2 - m_2^2)^2}{\frac{\vec{k}_\perp^2 + m_1^2}{x_1} + \frac{\vec{k}_\perp^2 + m_2^2}{x_2}} \right\}, \quad (29)$$

where $x_1 + x_2 = 1$; $m_{1,2}$ is the mass of valence quark. Setting $x_1 = x$, we can obtain

$$\phi(\vec{k}_\perp, x) = N \exp \left\{ \frac{-1}{8\beta^2} \left[\frac{\vec{k}_\perp^2 + \bar{x} m_1^2 + x m_2^2}{x \bar{x}} + \frac{(m_1^2 - m_2^2)^2 x \bar{x}}{\vec{k}_\perp^2 + \bar{x} m_1^2 + x m_2^2} \right] \right\}, \quad (30)$$

where N is a normalization factor.

Using the aforementioned convention, we get

$$A_0^{\Upsilon \rightarrow B_c}(0) = A_3^{\Upsilon \rightarrow B_c}(0) = 0.81 \pm 0.01, \quad (31)$$

$$A_1^{\Upsilon \rightarrow B_c}(0) = 0.83 \pm 0.01, \quad (32)$$

$$A_2^{\Upsilon \rightarrow B_c}(0) = 0.73 \pm 0.08, \quad (33)$$

$$V^{\Upsilon \rightarrow B_c}(0) = 1.98 \pm 0.06, \quad (34)$$

where the uncertainties come from variation of valence quark mass $m_{b,c}$. In addition, according to the NRQCD argument, the relativistic corrections and higher-twist effects might give uncertainties of $\mathcal{O}(v^2)$, about 10%~30%. Values at $q^2 \neq 0$ could, in principle, be extrapolated by assuming the form factors dominated by a proper pole which is unknown, or calculated with other method, such as the approach using the Bethe-Salpeter wave functions with the help of the nonrelativistic instantaneous approximation and the potential model based on the Mandelstam formalism [36] and so on. Here, we will follow the common practice for nonleptonic B decays with the QCDF approach. Values of form factors at $q^2 = 0$ are taken to offer an order of magnitude estimation, because both Υ and B_c are heavy quarkonium and the recoil effects might be not so significant.

D. Decay amplitudes

With the aforementioned definition of hadronic matrix elements, the decay amplitudes of $\Upsilon(1S) \rightarrow B_c M$ decays can be written as

$$\mathcal{A}(\Upsilon \rightarrow B_c^+ \pi^-) = \sqrt{2} G_F V_{cb} V_{ud}^* a_1 f_\pi m_\Upsilon (\epsilon_\Upsilon \cdot p_\pi) A_0^{\Upsilon \rightarrow B_c}, \quad (35)$$

$$\mathcal{A}(\Upsilon \rightarrow B_c^+ K^-) = \sqrt{2} G_F V_{cb} V_{us}^* a_1 f_K m_\Upsilon (\epsilon_\Upsilon \cdot p_K) A_0^{\Upsilon \rightarrow B_c}, \quad (36)$$

$$\begin{aligned} \mathcal{A}(\Upsilon \rightarrow B_c^+ \rho^-) = & -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 f_\rho m_\rho \left\{ (\epsilon_\Upsilon \cdot \epsilon_\rho^*) (m_\Upsilon + m_{B_c}) A_1^{\Upsilon \rightarrow B_c} \right. \\ & \left. + (\epsilon_\Upsilon \cdot p_\rho) (\epsilon_\rho^* \cdot p_\Upsilon) \frac{2 A_2^{\Upsilon \rightarrow B_c}}{m_\Upsilon + m_{B_c}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\Upsilon^\mu \epsilon_\rho^{*\nu} p_\Upsilon^\alpha p_\rho^\beta \frac{2 V^{\Upsilon \rightarrow B_c}}{m_\Upsilon + m_{B_c}} \right\}, \end{aligned} \quad (37)$$

$$\begin{aligned} \mathcal{A}(\Upsilon \rightarrow B_c^+ K^{*-}) = & -i \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* a_1 f_{K^*} m_{K^*} \left\{ (\epsilon_\Upsilon \cdot \epsilon_{K^*}^*) (m_\Upsilon + m_{B_c}) A_1^{\Upsilon \rightarrow B_c} \right. \\ & \left. + (\epsilon_\Upsilon \cdot p_{K^*}) (\epsilon_{K^*}^* \cdot p_\Upsilon) \frac{2 A_2^{\Upsilon \rightarrow B_c}}{m_\Upsilon + m_{B_c}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon_\Upsilon^\mu \epsilon_{K^*}^{*\nu} p_\Upsilon^\alpha p_{K^*}^\beta \frac{2 V^{\Upsilon \rightarrow B_c}}{m_\Upsilon + m_{B_c}} \right\}. \end{aligned} \quad (38)$$

For the $\Upsilon(1S) \rightarrow B_c V$ decays, the hadronic matrix elements in Eq.(8) can also be expressed as [37]

$$\begin{aligned} \mathcal{H}_\lambda = & \langle V | J^\mu | 0 \rangle \langle B_c | J_\mu | \Upsilon(1S) \rangle \\ = & \epsilon_V^{*\mu} \epsilon_\Upsilon^\nu \left\{ a g_{\mu\nu} + \frac{b}{m_\Upsilon m_V} (p_\Upsilon + p_{B_c})^\mu p_V^\nu + \frac{i c}{m_\Upsilon m_V} \epsilon_{\mu\nu\alpha\beta} p_V^\alpha (p_\Upsilon + p_{B_c})^\beta \right\}. \end{aligned} \quad (39)$$

The definition of helicity amplitudes is

$$\mathcal{H}_0 = -a y - 2b(y^2 - 1), \quad (40)$$

$$\mathcal{H}_\pm = a \pm 2c \sqrt{y^2 - 1}, \quad (41)$$

where invariant amplitudes a , b , c and variable y are

$$a = -i f_V m_V (m_\Upsilon + m_{B_c}) A_1^{\Upsilon \rightarrow B_c}(q^2), \quad (42)$$

$$b = -i f_V m_\Upsilon m_V^2 \frac{A_2^{\Upsilon \rightarrow B_c}(q^2)}{m_\Upsilon + m_{B_c}}, \quad (43)$$

$$c = +i f_V m_\Upsilon m_V^2 \frac{V^{\Upsilon \rightarrow B_c}(q^2)}{m_\Upsilon + m_{B_c}}, \quad (44)$$

$$y = \frac{p_\Upsilon \cdot p_V}{m_\Upsilon m_V} = \frac{m_\Upsilon^2 - m_{B_c}^2 + m_V^2}{2 m_\Upsilon m_V}. \quad (45)$$

The scalar amplitudes a , b , c describe the s , d , p wave contributions, respectively. Clearly, compared with the s wave amplitude, the p and d wave amplitudes are suppressed by a factor m_V/m_Υ .

III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of $\Upsilon(1S)$ particle, branching ratio for nonleptonic $\Upsilon(1S) \rightarrow B_c M$ weak decays can be written as

$$Br(\Upsilon(1S) \rightarrow B_c M) = \frac{1}{12\pi} \frac{p_{\text{cm}}}{m_\Upsilon^2 \Gamma_\Upsilon} |\mathcal{A}(\Upsilon(1S) \rightarrow B_c M)|^2, \quad (46)$$

where the decay width $\Gamma_\Upsilon = 54.02 \pm 1.25$ keV [4]; the momentum of final states is

$$p_{\text{cm}} = \frac{\sqrt{[m_\Upsilon^2 - (m_{B_c} + m_M)^2][m_\Upsilon^2 - (m_{B_c} - m_M)^2]}}{2 m_\Upsilon}. \quad (47)$$

The input parameters, including the CKM Wolfenstein parameters, masses of b and c quarks, decay constants, and Gegenbauer moments of distribution amplitudes in Eq.(7), are collected in Table III. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on the CP -averaged branching ratios for the $\Upsilon(1S) \rightarrow B_c M$ decays are displayed in Table IV, where theoretical uncertainties of the QCDF results come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.5)m_b$, masses of b and c quarks, hadronic parameters (decay constants and Gegenbauer moments), respectively. For the sake of comparison, previous results of Refs. [22, 23] are re-evaluated with coefficient $a_1 = 1.054$, where the scenario of the flavor dependent parameter ω in Ref. [23] is taken. The following are some comments.

(1) The QCDF's results fall in between those of Ref. [22] and Ref. [23], because the form factors $A_{0,1}^{\Upsilon \rightarrow B_c}$ in our calculation fall in between those of Ref. [22] and Ref. [23].

(2) There is a clear hierarchical relationship, $\mathcal{Br}(\Upsilon(1S) \rightarrow B_c \rho) > \mathcal{Br}(\Upsilon(1S) \rightarrow B_c \pi) > \mathcal{Br}(\Upsilon(1S) \rightarrow B_c K^*) > \mathcal{Br}(\Upsilon(1S) \rightarrow B_c K)$. These are two dynamical reasons. One is that the CKM factor $V_{cb}V_{us}^*$ responsible for the $\Upsilon(1S) \rightarrow B_c K^{(*)}$ decay is suppressed by a factor of λ relative to the CKM factor $V_{cb}V_{ud}^*$ responsible for the $\Upsilon(1S) \rightarrow B_c \pi, B_c \rho$ decays. The other is that the orbital angular momentum $L_{B_c P} > L_{B_c V}$.

(3) The CKM-favored a_1 dominated $\Upsilon(1S) \rightarrow B_c \rho$ decay has the largest branching ratio, $\sim 10^{-10}$, which should be sought for with high priority and firstly observed at the running LHC and forthcoming SuperKEKB. For example, the $\Upsilon(1S)$ production cross section in p-Pb collision can reach up to a few μb with the ALICE detector at LHC [38]. Therefore, per 100 fb^{-1} data collected at ALICE, over 10^{11} $\Upsilon(1S)$ particles are in principle available, corresponding to tens of $\Upsilon(1S) \rightarrow B_c \rho$ events if with about 10% reconstruction efficiency.

(4) There are many uncertainties on the QCDF's results. The first uncertainty, about 7~8%, from the CKM factors could be lessened with the improvement on the precision of the Wolfenstein parameter A . The second uncertainty from the renormalization scale should, in principle, be reduced by inclusion of higher order α_s corrections to hadronic matrix elements. The third uncertainty is due to the fact that masses of b and c quark affect the shape lines of wave functions, and hence the magnitude of form factors and branching ratios. The fourth uncertainty from hadronic parameters is expected to be reduced with the relative ratio of branching ratios.

(5) Other factors, such as the contributions of higher order corrections to hadronic matrix elements, relativistic effects, q^2 dependence of form factors and so on, which are not considered in this paper, deserve the dedicated study. Our results just provide an order of magnitude estimation.

IV. SUMMARY

With the sharp increase of the $\Upsilon(1S)$ data sample at high-luminosity dedicated heavy-flavor factories, the bottom-changing $\Upsilon(1S) \rightarrow B_c M$ weak decays are interesting in exploring the underlying mechanism responsible for transition between heavy quarkonia, investigating perturbative and nonperturbative effects and overconstraining parameters from B decays. The $\Upsilon(1S)$ weak decays are allowable within the standard model, though their branching ratios are expected to be tiny in comparison to the conventional strong and electromagnetic

TABLE III: The values of input parameters.

| Wolfenstein parameters | |
|---|-----------------------------------|
| $\lambda = 0.22537 \pm 0.00061$ [4] | $A = 0.814^{+0.023}_{-0.024}$ [4] |
| masses of quarks | |
| $m_c = 1.67 \pm 0.07$ GeV [4] | $m_b = 4.78 \pm 0.06$ GeV [4] |
| decay constants | |
| $f_\pi = 130.41 \pm 0.20$ MeV [4] | $f_K = 156.2 \pm 0.7$ MeV [4] |
| $f_\rho = 216 \pm 3$ MeV [32] | $f_{K^*} = 220 \pm 5$ MeV [32] |
| Gegenbauer moments at the scale $\mu = 1$ GeV | |
| $a_1^\rho = 0$ [32] | $a_2^\rho = 0.15 \pm 0.07$ [32] |
| $a_1^{K^*} = -0.03 \pm 0.02$ [32] | $a_2^{K^*} = 0.11 \pm 0.09$ [32] |
| $a_1^\pi = 0$ [33] | $a_2^\pi = 0.25 \pm 0.15$ [33] |
| $a_1^K = -0.06 \pm 0.03$ [33] | $a_2^K = 0.25 \pm 0.15$ [33] |

TABLE IV: The CP -averaged branching ratios for the $\Upsilon(1S) \rightarrow B_c M$ decays.

| | Ref. [22] | Ref. [23] | this work |
|--|-----------|-----------|--|
| $10^{10} \times \mathcal{Br}(\Upsilon(1S) \rightarrow B_c \rho)$ | 1.84 | 1.2 | $1.53^{+0.11+0.10+0.03+0.04}_{-0.10-0.04-0.04-0.04}$ |
| $10^{11} \times \mathcal{Br}(\Upsilon(1S) \rightarrow B_c \pi)$ | 6.91 | 2.8 | $5.03^{+0.36+0.34+0.09+0.02}_{-0.34-0.14-0.11-0.02}$ |
| $10^{12} \times \mathcal{Br}(\Upsilon(1S) \rightarrow B_c K^*)$ | 10.47 | 6.2 | $8.75^{+0.68+0.55+0.16+0.40}_{-0.64-0.20-0.22-0.39}$ |
| $10^{12} \times \mathcal{Br}(\Upsilon(1S) \rightarrow B_c K)$ | 5.03 | 2.3 | $3.73^{+0.29+0.25+0.07+0.03}_{-0.27-0.10-0.08-0.03}$ |

decays. In this paper, we studied the nonleptonic $\Upsilon(1S) \rightarrow B_c M$ weak decays, which are a_1 -dominated based on the low energy effective theory, and hence should have large branching ratios among weak decay modes. Considering the nonfactorizable contributions to hadronic matrix elements with the QCDF approach, we estimated the branching ratios of the $\Upsilon(1S) \rightarrow B_c M$ weak decays, where transition form factors are obtained by the integrals of wave functions with the nonrelativistic isotropic harmonic oscillator potential. The prediction on branching ratios for the $\Upsilon(1S) \rightarrow B_c M$ decays is the same order as previous works [22, 23]. The CKM favored $\Upsilon(1S) \rightarrow B_c \rho$ decay has relatively large branching ratio, $\sim 10^{-10}$, and might be detectable in future experiments.

Acknowledgments

We thank the referees for their helpful comments. The work is supported by the National Natural Science Foundation of China (Grant Nos. 11475055, 11275057, U1232101 and U1332103). Dr. Wang thanks for the support from CCNU-QLPL Innovation Fund (QLPL201411).

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